Midterm Exam Calculus 1

26 September 2011, 9.00-11.00.

Write on each page your name and student number, and on the first page your seminar group. The use of annotations, books and calculators is not permitted in this examination. All answers must be supported by arguments/work. Success.

1. (a) Formulate the principle of mathematical induction.

   (b) Prove with the help of the principle of mathematical induction that for every positive integer $n \geq 1$:

   $$1\,(1!) + 2\,(2!) + 3\,(3!) + \cdots + n\,(n!) = (n+1)! - 1$$

   Note: $n! = 1 \cdot 2 \cdot 3 \cdots n$.

2. Find all (complex) solutions of

   $$z^2 = \frac{1}{1+i}$$

   and plot them in the complex plane.

3. Determine all complex numbers z satisfying

   $$\mathrm{e}^z = 1 - i$$

4. The function $f(x)$ is defined for $-\infty < x < \infty$. In addition, we have

   $$\lim_{x \to 0} f(x) = 2.$$

   (a) Give the ϵ-δ-definition of $\lim_{x\to 0} f(x) = 2$.

   (b) Prove (using this definition) that a number $\delta > 0$ exists such that $f(x) > 1$ if $|x| < \delta$.

Maximum score:

1a	1.0	2	2.0	3	2.0	4a	1.0
b	2.0					b	1.0

Total: 9 + 1 (free) = 10.